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Effect of radial fins on two-dimensional turbulent natural convection in a horizontal annulus

Mohammad Rahnama *, Mousa Farhadi

Mechanical Engineering Department, Shahid Bahonar University of Kerman, Kerman, Iran

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Abstract

Turbulent natural convection between two horizontal concentric cylinders in presence of radial fins is studied numerically. Turbulence modeling was done using a standard k–ε model. The conservation equations of mass, momentum and energy along with modeled equations of turbulent kinetic energy and its dissipation rate were discretized using control-volume approach. Results are obtained in the form of streamline and temperature contours for a number of radial fins ranges from 2 to 12 which are attached to the inner cylinder. Two different configurations are investigated for the case of two and four radial fins to reveal the effect of fin height and fin configuration. The Rayleigh number considered in this study ranges from 10^6 to 10^9. Results obtained for local Nusselt number variation of the inner cylinder shows that fin arrangement has no significant effect on heat transfer and higher fin heights have a blocking effect on flow causing lower heat transfer rate. It is observed that there is a reduction of heat transfer rate in all of the configurations considered in this study as compared to the case of no fin at the same Rayleigh number. There is also an increase in Nusselt number prediction with increasing Rayleigh number, which is a general behavior for all of natural convection problems.
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Keywords: Natural convection; Annulus; Fin; Turbulent; Finite volume

1. Introduction

Natural convection between two horizontal concentric cylinders has received much attention because of certain theoretical concerns and experimental applications. From theoretical point of view, different density stratification in the upper and lower portions of domain causes several flow pattern and regimes to exist in different regions of annulus which in turn, makes accurate prediction of flow and temperature fields more difficult. Engineering application of this flow such as inert gas insulation of high electric cables, receivers of some focusing solar collector and thermal energy storage systems reveals the importance of such investigation. The first extensive research on this flow geometry was done by Kuhen and Goldstein [1]. In their work, an experimental and numerical investigation has been carried out for water and air in annulus for gap width to inner cylinder diameter of 2.6. Rayleigh number based on the gap width was 10^4 to 10^5. In their experimental study, using air as a working fluid, it was reported that transition from laminar to turbulent flow occurs at Rayleigh number equal to 10^6.

Turbulent flow regime exists for high Rayleigh numbers. The critical Rayleigh number depends on the inner to outer cylinder diameter ratio. Some papers were published investigating turbulent natural convection between two concentric cylinders. Kuhen and Goldstein [2] measured heat transfer coefficients for air and water in concentric and eccentric horizontal annuli for Rayleigh numbers up to 10^7. Bishop [3] reported an experimental investigation of turbulent natural convection of helium between horizontal isothermal concentric cylinders at cryogenic temperature. With the next experimental work of Mcleod and Bishop [4] for $\mathrm{RR} = 4.85$ they concluded that the heat transfer rate depends on magnitude of the expansion number as well as on the Rayleigh number.

Large eddy simulation of turbulent natural convection between concentric horizontal cylinders was done by Miki et al. [5]. The highest Rayleigh number for which solution obtained, was 1.18×10^9. The values of mean Nusselt number were in good agreement with those obtained from experiments, except for the highest Rayleigh number. Desai and Vafai [6] employing finite element method carried out

* Corresponding author.
E-mail addresses: rahnama@mail.uk.ac.ir (M. Rahnama), farhadi@mail.uk.ac.ir (M. Farhadi).

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Nomenclature

g	gravity acceleration	$\mathrm{m{\cdot}s^{-2}}$
H	fin height	m
k	Von Driest constant	
K	turbulent kinetic energy	$\mathrm{m^2{\cdot}s^{-2}}$
K_{eq}	equivalent thermal conductivity, $= q_{\mathrm{act}}/q_{\mathrm{cond}}$	
l	gap width, $= R_o - R_i$	m
n	normal vector on the wall	
Nu	local Nusselt number	
$\overline{Nu}$	mean Nusselt number, $= \frac{1}{\pi R}\int_0^{\pi}(Nu)r\,\mathrm{d}\theta$	
$\overline{Nu_o}$	mean Nusselt number without fin, $= \frac{1}{\pi R}\int_0^{\pi}(Nu)r\,\mathrm{d}\theta$	
P	pressure	Pa
Pr	Prandtl number	
R	radius of cylinders	m
r	radial coordinate	
Ra	Rayleight number, $= g\beta l^3(T_i - T_o)/\nu\alpha$	
RR	radius ratio, $= R_o/R_i$	
T	temperature	K
u^+	dimensionless value of u, $= u/\sqrt{\tau_w/\rho}$	
V	velocity	$\mathrm{m{\cdot}s^{-1}}$
y	distance from the wall	
y^+	dimensionless value of y, $= y\sqrt{\tau_w/\rho}/\upsilon$	

Greek letters

ρ	density	$\mathrm{kg{\cdot}m^{-3}}$
ν	kinematic viscosity	$\mathrm{m^2{\cdot}s^{-1}}$
ε	turbulent energy dissipation rate	$\mathrm{m^2{\cdot}s^{-3}}$
μ	dynamic viscosity	$\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-1}}$
λ	thermal conductivity	$\mathrm{W{\cdot}m^{-1}{\cdot}{}^{\circ}C^{-1}}$
σ	turbulent Prandtl number	
β	volumetric coefficient of thermal expansion	$\mathrm{K^{-1}}$
τ	shear stress	$\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-2}}$
Δ	height of the wall element	

Superscript

$*$	star denotes dimensionless parameters

Subscripts

t	turbulent characteristic
eff	effective
r	radial direction
θ	tangential direction
o	outer
i	inner
act	actual
cond	conduction
w	wall

a comprehensive work with detailed analysis of two- and three-dimensional turbulent natural convection in a horizontal annulus. The standard k–ε turbulence model using a wall function approach was used for modeling turbulent flow. They showed that if the annulus is sufficiently long, there exists a core region over a substantial length of the cavity that can be approximated by a two-dimensional model. Recently, a number of papers on numerical prediction of turbulent natural convection in a concentric horizontal annulus were published which incorporated the effect of inner cylinder rotation [7], multiple perturbation [8], transition of natural convection in a horizontal annulus [9] and very small radius difference [10].

The most important parameter of interest in natural convection between two concentric cylinders is the rate of heat transfer which is constant for a prescribe boundary conditions and flow geometry. An essential restriction in natural convection in annulus is heat transfer limitation due to the fixed area of the inner and outer cylinders. One approach affecting heat transfer rate in annulus is application of some radial fins on inner cylinder. Existence of such fins makes the flow and heat transfer characteristics substantially different from those of annulus without fins. This flow geometry has been investigated for laminar flow case by Chai and Patankar [11] and Rahnama et al. [12]. Chai and Patankar [11] studied the flow and heat transfer for an annulus with six radial fins arranged for two different configurations. It was observed that orientation of the internal fins has no important effect on the average Nusselt number prediction, while the blockage due to the fins has significant effect on the flow and temperature fields and therefore on heat transfer rate. Rahnama et al. [12] examined laminar natural convection in annular region numerically.

Detailed data on natural convection in horizontal concentric cylinders with radial fins are very limited. Recently Abu-Hijleh [13] studied laminar natural convection and entropy generation from a horizontal cylinder with multiple, equally spaced, low conductivity baffles on its outer surface. It was shown that short baffles slightly increases the heat transfer rate at small values of Rayleigh numbers while there was up to 72 percent reduction in the value of the average Nusselt number depending on the number and height of baffles as well as the Rayleigh number.

To the best of author's knowledge, there are no published data investigating the effect of radial fins on heat transfer in turbulent natural convection in annular geometry. In this study, the effect of radial fins on turbulent flow pattern, temperature distribution and Nusselt number are presented for various fin arrangements and different number of fins. The main purpose of the present study is to investigate the effect of fins on heat transfer in turbulent natural convection.

2. Governing equations

The configurations to be studied are shown in Figs. 1 and 2. Each of the configurations of Fig. 2 consists of two cylindrical surfaces and a number of fins attached to the inner cylinder surface. Each fin has a very small thickness which is equal to one grid cell in tangential direction. Cylindrical coordinates (r, θ) was used for the present computation with $\theta = 0$ at the bottom of the annulus. The thermo physical properties of fluid are assumed constant except density for which the Boussinesq approximation is employed such that the variation of density with temperature has been neglected except for buoyancy force terms. The viscous dissipation in energy equation does not need to be considered because of low fluid velocity in natural convection.

The basic equations governing fluid flow and heat transfer in annular geometry are the laws of conservation of mass, momentum and energy, put into the non-dimensional form by taking the characteristic length, velocity, pressure and temperature as 1 (radius difference), $V = \nu/1$, $\rho_0 V^2$ and $(T_i - T_o)$. The time-averaged Reynolds equations governing turbulent natural convection in the annular enclosure are written as:

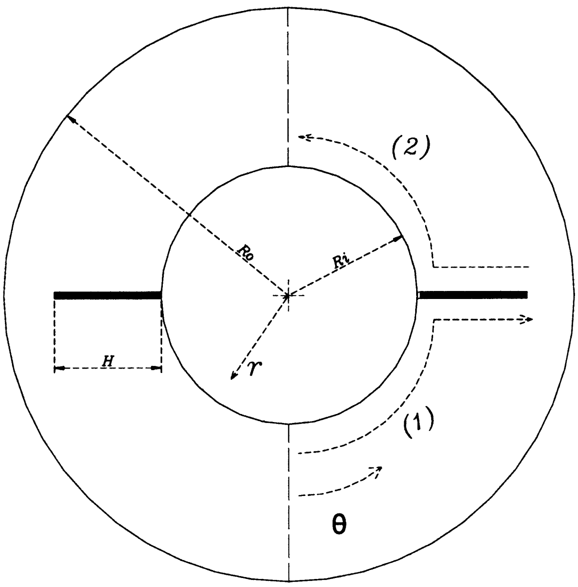


Fig. 1. Flow geometry for annulus with fins.

Continuity:

$$V^*_{j,j} = 0 \tag{1}$$

Momentum:

$$V^*_j V^*_{i,j} = -P^*_{,i} + \left[\mu_{\text{eff}}\left(V^*_{i,j} + V^*_{j,i}\right)\right]_{,j} - \frac{Ra}{Pr} T^* \tag{2}$$

Energy:

$$V^*_j T^*_{,j} = \frac{1}{Pr}\left[\lambda^*_{\text{eff}} T^*_{,j}\right]_{,j} \tag{3}$$

Turbulent kinetic energy:

$$V^*_j K^*_{,j} = \left[\frac{\mu^*_t}{\sigma_K} K^*_{,j}\right]_{,j} = \frac{Ra}{Pr}\frac{\mu^*_t}{\sigma_t} T^*_{,j} - \varepsilon^* + \mu^*_t\left[V^*_{i,j} + V^*_{j,i}\right]V^*_{i,j} \tag{4}$$

Dissipation of turbulent kinetic energy:

$$V^*_j \varepsilon^*_{,j} = \left[\frac{\mu^*_t}{\sigma_\varepsilon}\varepsilon^*_{,j}\right]_{,j} - C_3\frac{Ra}{Pr}\frac{\mu^*_t \varepsilon^*}{\sigma_t K^*} T^*_{,j} - C_2\frac{\varepsilon^{*2}}{K^*} + C_1\frac{\varepsilon^*}{K^*}\mu^*_t\left[V^*_{i,j} + V^*_{j,i}\right]V^*_{i,j} \tag{5}$$

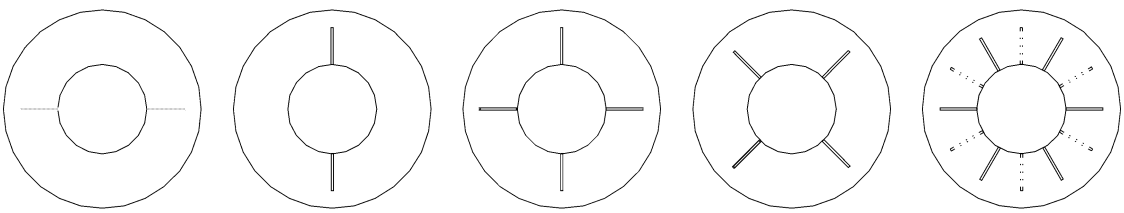
In the above equation effective viscosity, μ^*_{eff}, and effective thermal diffusivity λ^*_{eff}, are defined as:

$$\mu^*_{\text{eff}} = 1 + \mu^*_t \quad \text{and} \quad \lambda^*_{\text{eff}} = 1 + \lambda^*_t \tag{6}$$

where:

$$\begin{aligned} &r^* = \frac{r}{l}, \quad V^*_r = \frac{lV_r}{\nu}, \quad V^*_\theta = \frac{lV_\theta}{\nu} \\ &K^* = \frac{l^2K}{\nu^2}, \quad \varepsilon^* = \frac{l^4\varepsilon}{\nu^3}, \quad \lambda^*_t = \frac{\mu^*_t}{Pr_t}Pr \\ &P^* = \frac{l^2P}{\rho\nu^2}, \quad \mu^*_t = \frac{\mu_t}{\mu}, \quad T^* = \frac{T - T_o}{T_i - T_o} \\ &\mu_t = C_\mu\frac{K^2}{\varepsilon} \end{aligned} \tag{7}$$

The values of the constants that appear in the governing equations are: $C_1 = 1.44$, $C_2 = 1.92$, $C_\mu = 0.09$, $C_3 = 1.44$, $Pr = 1.0$, $\sigma_\kappa = 1.0$ and $\sigma_\varepsilon = 1.3$. Although these constants are well-established from data obtained for turbulent forced convection flows, their use for turbulent natural convection is justified as it is shown by Desai and Vafai [6]. They mentioned that variation of these constants has a negligible influence on the results. The standard k–ε turbulence model used in this study is suitable for high Rayleigh number flows which occur in the core region of the annulus. The resolution of sharp gradient of flow variables in the near wall and solid surfaces of the ribs cannot be achieved with



(a) (b) (c) (d) (e)

Fig. 2. Various configuration fin arrangements.

standard k–ε turbulence model. Another shortcoming is the inability of standard model to resolve the effect of viscosity on turbulence field in the viscous sub layer. To overcome these shortcomings a scheme was used for wall treatment which is discussed in detail by Ciofalo and Collins [14]. In this scheme which is used in the present work, the fully turbulent outer flow field and the physical boundary are "bridged" by using a single layer of specialized wall elements. The interpolation functions in these wall elements are based on universal near-wall profiles. They are functions of the characteristic turbulent Reynolds numbers which accurately resolve the local flow and temperature profiles. The turbulent diffusivity in the near-wall region is calculated using Van Driest's mixing length approach. The standard k–ε equations are solved in the part of the computational domain excluding the wall element region. The elliptic form of the mean conservation equations are solved throughout the computational domain. However, the k–ε model is applied only up to and excluding the wall elements. The application of the present approach to complicated forced flows involving strong and subtle flow reversal has already been demonstrated by Haroutunian and Engelman [15]. The resulting model was found to be more accurate and computationally more effective than the k–ε model using standard wall functions. A brief description of this model is given below. As mentioned before, the viscosity-affected region between the wall and the fully turbulent region away from the wall is represented by means of a single layer of special elements. These specialized shape functions which are based on universal near-wall velocity profiles accurately resolve the velocity profiles near the wall. The functional form used for the wall element is:

$$\begin{aligned} u^+ &= f_R(y^+) \\ &= \frac{1}{\kappa}\ln(1+0.4y^+) \\ &\quad + 7.8\left[1-\exp\left(-\frac{y^+}{11}\right)-\frac{y^+}{11}\exp(-0.33y^+)\right] \end{aligned} \tag{8}$$

The universal profile used for temperature takes the following form:

$$\begin{aligned} T^+ &= f_T(y^+, Pr, Pr_t) \\ &= Pr D_T u^+ + Pr_t(1-D_T)(u^+ + P_T) \end{aligned} \tag{9}$$

where y^+ is a dimensionless distance from the wall and:

$$\begin{aligned} &D_T = \exp\left(-\frac{y^+}{11}R_T^a\right): \quad a = \begin{cases} 1.1 \rightarrow R_T < 0.1 \\ 0.333 \rightarrow R_T \geqslant 0.1 \end{cases} \\ &R_T = \frac{Pr}{Pr_t} \end{aligned} \tag{10}$$

And P_T based on the Jayatilleke correlation [16] is given by:

$$P_T = 9.24\left(R_T^{0.75}-1\right)\left[1+0.28\exp(-0.007R_T)\right] \tag{11}$$

The interpolation functions in the specialized elements are constructed using a tensor product of the one-dimensional basis functions corresponding to the local coordinate directions in the elements. The basis functions in the direction along the wall are the same as the elements in the rest of the computational domain. In the direction normal to the wall, the interpolation functions are based on Eqs. (8)–(11). The boundary conditions for this problem with the heated inner cylinder can be given as: no-slip conditions for the velocity components at the boundary, uniform but different temperatures of the inner and outer cylinders, uniform temperature equal to inner cylinder surface temperature for fin surfaces (or fins with high thermal conductivity) and vanishing of pressure gradient, turbulent kinetic energy and dissipation rate on the boundaries. The boundary conditions for K and ε the first grid point away from the wall are:

$$\left.\begin{aligned} \frac{\partial K}{\partial n} &= 0 \\ \varepsilon_p &= \frac{(C_\mu K)^{1.5}}{\kappa\Delta} \end{aligned}\right\} \quad \text{At the first grid point away from the wall} \tag{12}$$

This boundary condition for K plays an important role in the approach used here. It allows the value of K to adjust in response to the turbulent processes in both local and neighboring regions.

3. Solution procedure

The governing differential equations for mass, momentum, energy, turbulent kinetic energy and its dissipation rate were solved using the control-volume-based finite difference method described by Patankar [17]. The convective and diffusive terms were discretized using power law scheme and the SIMPLER algorithm [17] was used to resolve the pressure-velocity coupling. The grid points are not distributed uniformly over the computational domain. They have greater density near the surface of the cylinders in the radial direction and near the fins in the tangential direction and have a lower density in the free region between the surface and the fin. The expansion and contraction factors were selected 1.05 and 0.95, respectively. Selections of the number of grid points used in the present computation are reasoned based on Nusselt number prediction along the inner and outer cylinders. Fig. 3 represents the results of the Nusselt number variation in tangential direction along the inner cylinder for annulus without fin. This figure shows that

Table 1
Effect of grid points on mean Nusselt number for geometry of Fig. 2(a)

Grid points	40×40	60×60	70×70	60×80	80×80
$\overline{Nu}$	4.303	5.01	5.072	5.021	5.09
Percent of error	14.11%	0.0%	1.23%	0.22%	1.59%

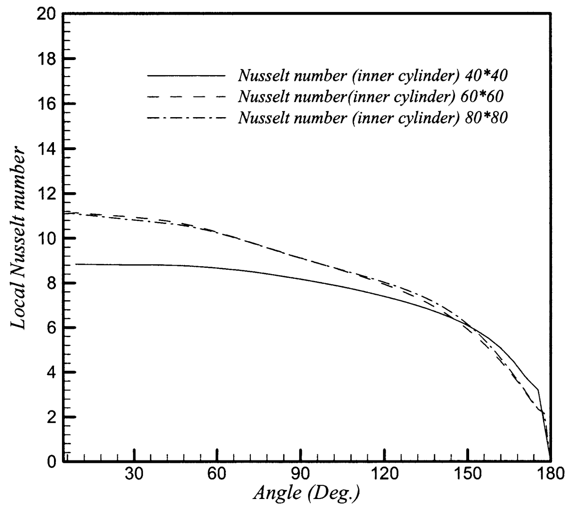


Fig. 3. Local Nusselt number variation on the inner cylinder, effect of number of grids.

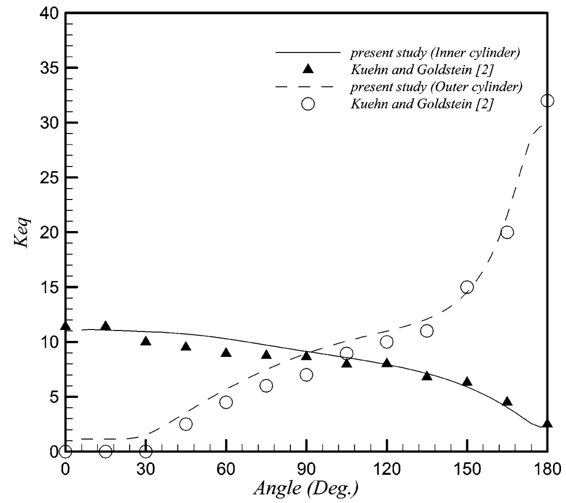


Fig. 4. Equivalent thermal conductivity on inner and outer cylinders for annulus without fin at $Ra = 2.51 \times 10^6$.

the Nusselt number prediction along the inner cylinder for 60×60 grid points nearly correspond to that obtained from 80×80 grid points, while its difference with 40×40 grid points solution is remarkable. Therefore the number of grid points is selected as 60×60 for the present computations. The effect of grid points on mean Nusselt number computation of annulus with two horizontal fins was investigated as well. Table 1 shows the effect of number of grid points on mean Nusselt number for geometry shown in Fig. 2(a). As is observed from this table, the difference between mean Nusselt number obtained for 60×60 grid points with that computed with other number of grid points is negligible (percent of error less than 1.6) except for 40×40 grid points. So the minimum number of grid points which produces acceptable results is 60×60.

4. Results and discussion

As mentioned before, Fig. 1 shows the geometry considered in this study. Fig. 2 represents different fin arrangements investigated in this paper. The first one is an annulus with two horizontal and vertical fins, Fig. 2(a) and (b), respectively. In the second geometry, the number of fins is selected as four, but their arrangements are chosen differently (Fig. 2(c) and (d)) to detect the effect of fin orientation on flow and temperature fields. Fig. 2(e) shows geometry of annulus with other number of fins. The maximum number of fins considered in this study is 12.

Results are represented in two parts. In the first part the accuracy of solution method is investigated by computing natural convection in annular region without fin. This configuration had been studied in detail by various authors and established to an extent that is used as a source of comparison for validating relevant numerical codes. The reason for doing such computations was to show the reliability of computational model in this geometry. In the second part, results are presented for turbulent natural convection in annulus with radial fins for different fin arrangement. The effect of fin height and Rayleigh number on flow and temperature field are also investigated.

All computations in this study were carried out for radius ratio of 2.6 and air ($Pr = 0.7$) as the working fluid. Dimensionless temperatures for the inner and outer cylinder are selected as one and zero, respectively. These values were chosen so as to facilitate comparison with the available experimental data. While the effect of fin height has studied for two-fin configuration, fin height is selected fixed as 20% of the radius difference for all other geometries. As mentioned in following sections, this was due to the relatively negligible effect of fin height on mean Nusselt number prediction for the geometry of two fins. Computations were performed for Rayleigh number ranges from 10^6 to 10^9 for all geometries.

4.1. Validation

The problem of natural convection between two concentric horizontal cylinders is studied both experimentally and numerically by Kuhen and Goldstein [2]. Computations were performed for this geometry to validate the accuracy of the numerical results by comparing with the experimental results [2]. An equivalent thermal conductivity, K_{eq} is used to compare the accuracy of the present computations. This parameter is defined as the actual heat flux divided by the heat flux that would be occurred by pure conduction in the absence of fluid motion. Results are shown in Fig. 4 and represent good agreement between the present computations and experiments of Kuhen and Goldstein [2].

4.2. Fluid flow

This section represents the results obtained for annulus with radial fins which are the main part of the present study. As mentioned before, different fin configurations were studied in this investigation (Fig. 2). The first geometry considered is an annulus with two fins attached to the inner cylinder. These fins could be in a horizontal (Fig. 2(a)) or vertical (Fig. 2(b)) direction. Fig. 5 shows streamline and temperature contours for each configuration at $Ra = 10^6$ and fin height to radius difference ratio of 0.2. It is observed that the center of recirculation zone has slightly moved to lower portion of annulus for vertical fin arrangements compared to the horizontal one. This effect makes little changes in temperature fields. But there is no significant difference in flow pattern for both geometries. Results of turbulent kinetic energy and its dissipation rate computations have shown that there is a high turbulence level near the center of recirculation toward the top of the outer cylinder while energy dissipation is concentrated near the rib and the surface of inner and outer cylinders. Investigation of eddy viscosity contours shows that there is a maximum near the center of recirculation region indicating that the Reynolds stress assumes a large value in this region. Therefore it can be concluded that annulus with two vertical fins experiences higher turbulent kinetic energy and Reynolds stress along with lower dissipation rate than the one with two horizontal fins. This behavior affects heat transfer rate as discussed in the next section.

Effect of fin height on flow and temperature fields has been studied for two fin arrangements. Rahnama and Farhadi [18] reported this effect on turbulent natural convection for the case of two horizontal fin arrangements. The streamline pattern obtained showed that increasing fin height to radius difference ratio more than 0.4 results in two recirculation zones. Its effect on Nusselt number variation will be discussed in the next subsection. As the aim of this paper was to investigate the effect of number of fins, the effect of fin height were limited to the case of two fins and the ratio of fin height to radius difference has selected as 0.2 for all other geometries.

The second geometry studied in this research, is annulus with four fins attached to the inner cylinder, Fig. 2(c) and (d). These two configurations have different fin arrangements. Fig. 2(c) shows an arrangement with two fins in horizontal and two fins in vertical direction while Fig. 2(d) shows the same fins rotate 45 degrees relative to horizontal and vertical arrangement. Flow and temperature fields in the form of streamline and isotherm are shown in Fig. 6. There are differences between the streamline and temperature contours for different configurations. Results represent remarkable differences in contours especially in the upper part of the annulus. The center of recirculation is near $\theta = 90$ degrees with a weak recirculation zone near the top rib for the case of Fig. 2(d) while this center is near the top for the case of

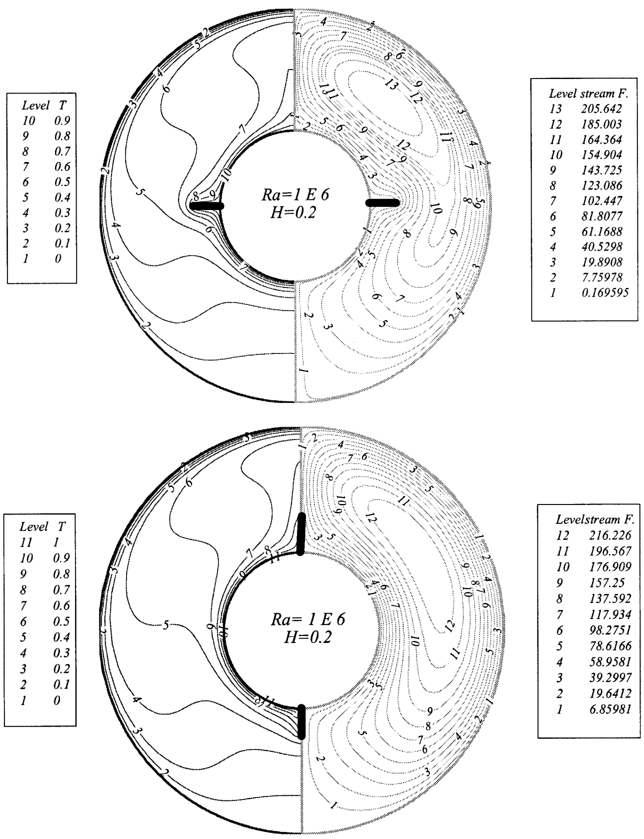


Fig. 5. Streamline and temperature contours for two (a) horizontal, and (b) vertical fins at $Ra = 10^6$.

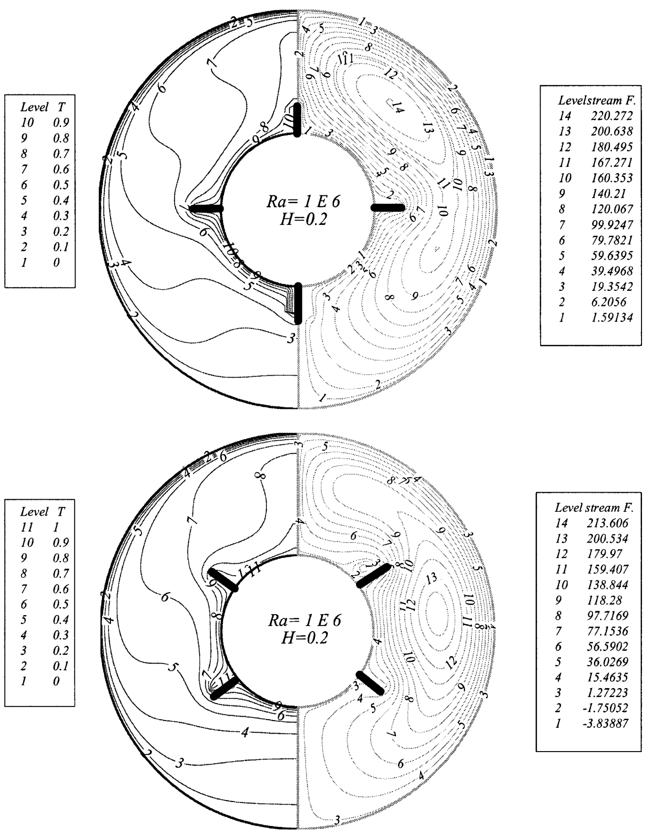


Fig. 6. Streamline and temperature contours for two different four-fin arrangements, both for $Ra = 10^6$.

Fig. 2(c). Computational results showed that there are two recirculation zones for lower Rayleigh numbers for the case shown in Fig. 2(c) which changes to one zone for higher Rayleigh numbers. The differences observed in Fig. 6 can be explained based on the following main points. Two vertical fins of the four fin arrangements in geometry of Fig. 2(c) are parallel with the fluid motion and therefore make lower resistance to the motion of fluid. Buoyancy-induced motion in the lower part of the annulus is very weak due to the fact that high temperature surface (inner cylinder surface) is over the low temperature one (outer cylinder surface). So there is a stable density stratification which resists to fluid flow. In fact, there is a very weak fluid motion in the lower part of the annulus. The lower fins shown in Fig. 2(d) make a high resistance to this weak fluid motion and nearly block it. This phenomenon does not happen for arrangement of Fig. 2(c).

The fluid flow being started to circulate in the upper part of annulus, may have enough momentum to continue its motion to the lower part. The fluid along the inner cylinder tends to move away from the symmetry line and tries to bend downward upon meeting the first fin. However, the buoyancy force created by heating of air prevents air from moving downward and thus creates a region of almost stagnant flow in the bottom of the annulus. Results of computation of energy dissipation and turbulent kinetic energy show that although each of these two is not the same for these two different four-fin arrangements, mean Nusselt number results which are discussed in the following section, reveal that the fin arrangements have no significant effect on it. Results of streamline and temperature contours for annulus with more than four fins were similar to the one with four fins, so they are not presented here. It should be mentioned that increasing the number of fins from 8 to 12 makes the number of recirculation regions to increase from one to two. Also, the value of y^+ for the first grid point near the surface was less than 5, i.e., in laminar sub layer in all computations.

4.3. Nusselt number

While flow and temperature fields in the form of streamline and temperature contours do not differ significantly for various fin arrangements, results of Nusselt number prediction along the inner cylinder show obvious differences with Rayleigh number. The main objective in this paper is to show the effect of fins on heat transfer in the form of Nusselt number as compared to the case of no fins. Heat transfer results are presented in terms of the Nusselt number which is defined as the ratio of the actual heat transfer to the pure conduction heat transfer rate. Thus the expressions for the Nusselt number over the inner and outer cylinders, respectively, are given by:

$$Nu_i = R_i \ln\left(\frac{R_o}{R_i}\right)\frac{\partial T}{\partial r}\bigg|_{r=R_i} \tag{13}$$

$$Nu_o = R_o \ln\left(\frac{R_o}{R_i}\right)\frac{\partial T}{\partial r}\bigg|_{r=R_o} \tag{14}$$

where $\partial T/\partial r$ temperature gradient in radial direction. To compute Nusselt number along the fin surfaces, this temperature gradient should be replaced with $\partial T/r\partial\theta$. A second-order accurate three-point differencing scheme was used to calculate the Nusselt number results. The mean Nusselt number is obtained by integrating the local Nusselt number over the inner cylinder and fin surfaces. To illustrate the effect of fins on heat transfer rate more clearly as compared to the case of no fins, all mean Nusselt number obtained for the case of annulus with fins, were divided by that computed for no fin at the same Rayleigh number. Fig. 7 shows local Nusselt number variation in tangential direction for regions 1 and 2 shown in Fig. 1. For the case of two horizontal fins, Fig. 7(a), it is observed that Nusselt number starts from a local maximum at $\theta = 0$ (bottom of the annulus) and decreases to nearly zero at the base of the fin at $\theta = 90$ degrees. The maximum temperature difference between cold fluid and hot surface of the inner cylinder at $\theta = 0$ causes maximum Nussel number at this point. Deceleration of flow and lower temperature difference between cylinder surface and adjacent fluid due to heating of the fluid makes the Nusselt number to decrease toward the point A in Fig. 7(a). The first minimum Nusselt number occurs at the base of the fin where there is really stagnant fluid. Nusselt number values over the fin surface are much lower than that over the surface of the cylinder. There is an increase and decrease of Nusselt number value from point B (refer to Fig. 1) to the point with $\theta = 180$ degrees.

The behavior of the local Nusselt number for the case of two vertical fins, Fig. 7(b), is different from that of horizontal ones. Here Nusselt number starts from a higher value for top of the rib at the bottom ($\theta = 0$) and decreases much faster to nearly zero at the base of the fin which is at $\theta = 0$ as well. From the base of the lower fin to the base of the upper fin there is an increase and decrease in Nusselt number prediction until $\theta = 180$ degrees. As compared to Fig. 7(a), Nusselt number increases to a higher maximum along the surface of the inner cylinder for vertical fins arrangement than that of horizontal fins. It is expected that mean Nusselt number gets higher values for the case of two vertical fins compared with two horizontal ones. Dependence of Nusselt number on Rayleigh number is such that increasing Rayleigh number increases Nusselt number significantly for $10^6 < Ra < 10^8$ but small increase in Nusselt number would be obtained if Rayleigh number changes from 10^8 to 10^9.

Fig. 8 shows variation of mean Nusselt number ratio for the case of two fin arrangements with ratio of fin height to radius difference. As mentioned before, the abscissa in this figure is the ratio of mean Nusselt number for the case of fins attached to the inner cylinder surface to the one obtained for annulus without any fin. This ratio is less than one for any fin height, so attaching two fins to the surface of the cylinder makes heat transfer rate to decrease no matter how the arrangement selected (horizontal or vertical). This behavior is observed in all configurations studied in this research. As is observed from this figure, increasing

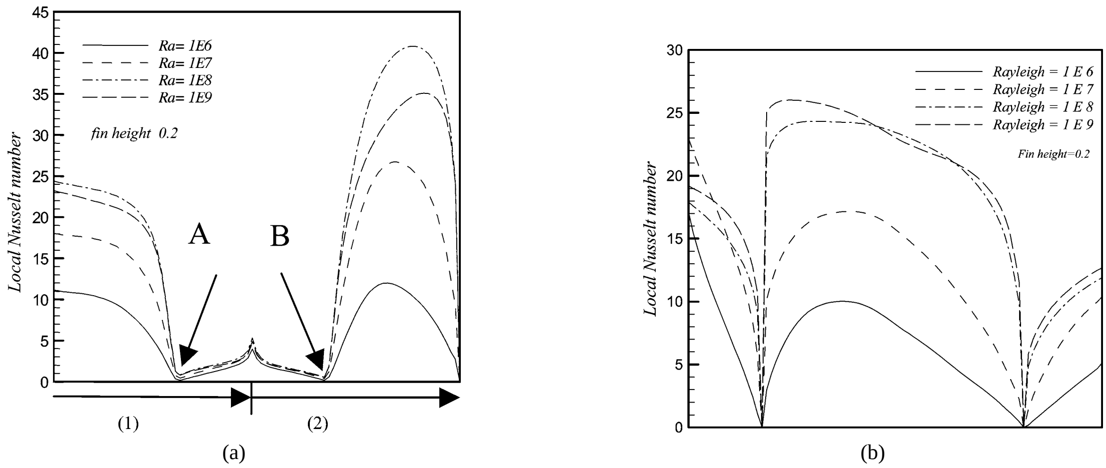


Fig. 7. Local Nusselt number variation for the case of two fin placed (a) horizontally, and (b) vertically for Rayleigh number of 10^6 to 10^9 and $H = 0.2$.

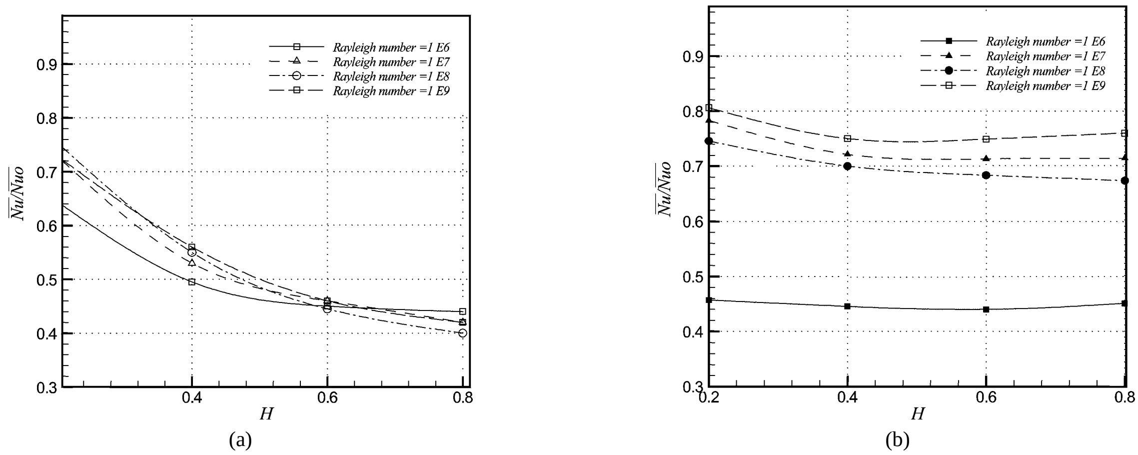


Fig. 8. Mean Nusselt number ratio various for fin height (a) horizontally, (b) vertically for two fins and Rayleigh number of 10^6 to 10^9.

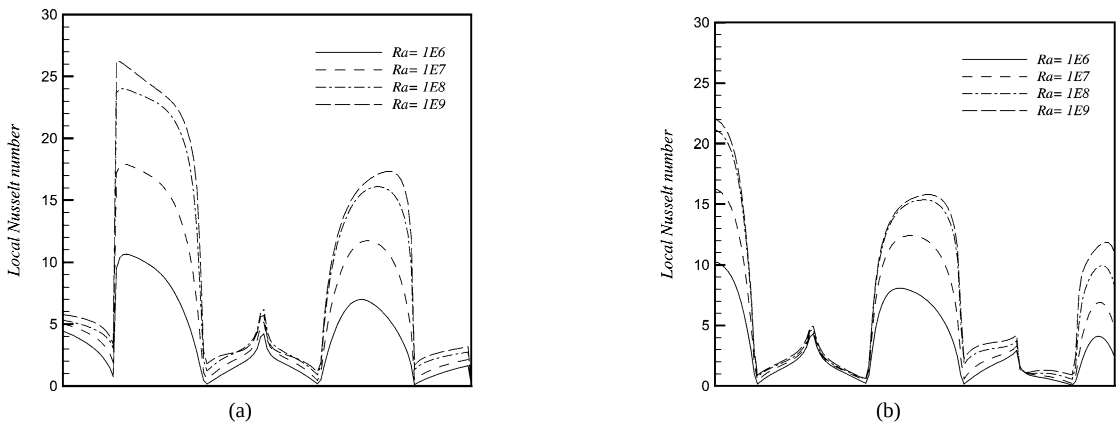


Fig. 9. Local Nusselt number variation for the case of four fin as shown in (a) Fig. 2(c); and (b) Fig. 2(d) for Rayleigh number of 10^6 to 10^9 and $H = 0.2$.

fin height decreases mean Nusselt number ratio. Therefore it is concluded that increasing fin height has a lowering effect on mean Nusselt number. As the rate of heat transfer is of interest in this geometry and increasing fin height increases heat transfer area, computations were performed for the ratio of heat transfer rate to that of pure conduction to see if heat transfer rate increases with increasing fin height. The ratio of heat transfer rate computed for fin height ratios of 0.2, 0.4 and 0.8 were 15.71, 14.56 and 11.74, respectively, for $Ra = 10^6$, which shows reduction of heat transfer rate with increasing fin height. This value for the case of no fin is 26.8. Comparing Fig. 8(a) with (b) shows that Nusselt number ratio dependence on fin height is more distinguishable for the case of two horizontal fins than that

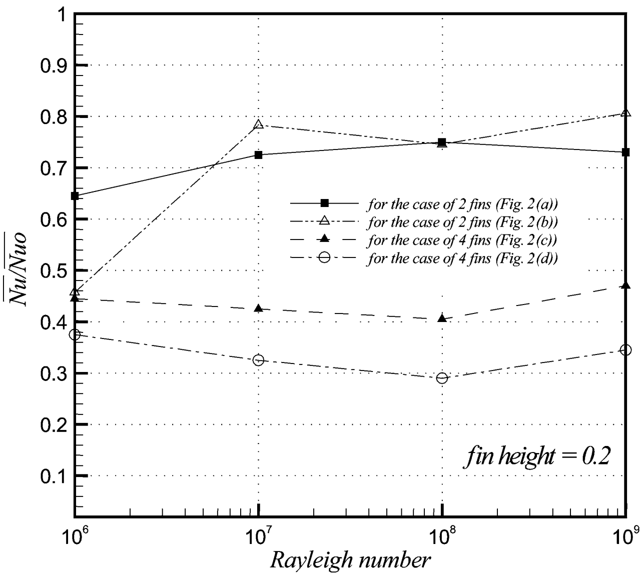


Fig. 10. Mean Nusselt number ratio various for Rayleigh number for fin height 0.2 and several fin arrangement.

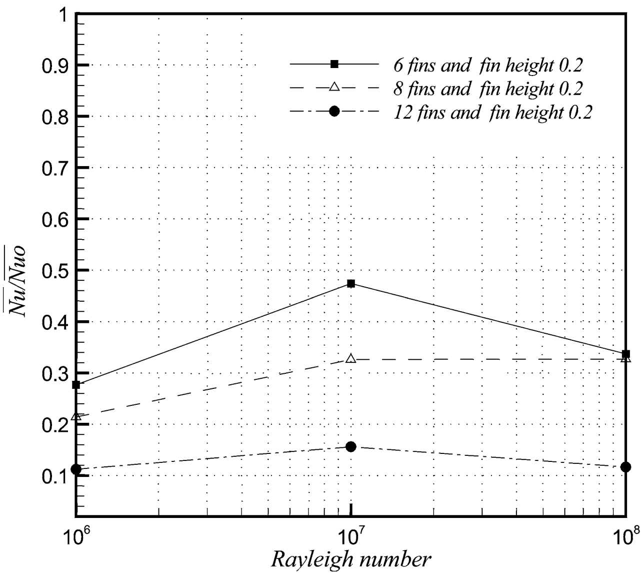


Fig. 11. Mean Nusselt number ratio various for Rayleight number for 6 to 12 fins at fin height 0.2.

of two vertical fins. Another observation in this figure is variation of Nusselt number ratio with Rayleigh number. This parameter increases with increasing Rayleigh number for both geometries. In most of the natural convection flows, there is an increase in Nusselt number with Rayleigh number.

Variation of local Nusselt number with angular position along the inner cylinder for the case of four-fin arrangements is seen in Fig. 9. The qualitative behavior of Nusselt number in this geometry is similar to that of two fin arrangements. However comparison of Figs. 9(a) and 7(a) shows a decrease in maximum Nusselt number for four-fin relative to two-fin arrangements which in fact shows the existence of lower temperature gradient on the surface of inner cylinder for four-fin arrangement. One reason for such behavior may be due to the lower fluid velocity for four-fin arrangements compare to two-fin arrangements. The effect of fin arrangement on mean Nusselt number is shown in Fig. 10. The variation of mean Nusselt number with Rayleigh number is negligible for four fin arrangements while its value is lower for the case of Fig. 2(d) than that of Fig. 2(c). The difference between mean Nusselt numbers for these two geometries is not remarkable. The results for two-fins arrangement is shown in this figure as well. It is observed that there are no differences in mean Nusselt number computation for Rayleigh numbers in the range of 10^6 to 10^9 for the case of two-fins arrangements.

The overall behavior of the local Nusselt number variation for 6-, 8- and 12-fins arrangements is similar to that of 4-fins and 2-fins arrangements; that is reduction of Nusselt number along the fins and alternate increasing and decreasing of Nusselt number between the fins. It is expected that reduction of mean Nusselt number ratio would be much higher for these number of fins compared to the case of four- and two-fins arrangements. This is observed in Fig. 11 where mean Nusselt number ratio is plotted versus Rayleigh number for different number of fins. As it is shown in this figure, mean Nusselt number ratio decreases with increasing number of fins with a slight dependence on Rayleigh number for each number of fins configurations.

5. Conclusion

In this paper, turbulent natural convection between two horizontal concentric cylinders with radial fins is studied numerically. The number of fins was selected between 2 to 12 and Rayleigh number was changed from 10^6 to 10^9. The following major conclusions can be drawn from this study:

(1) Fin arrangement has not significant effect on mean Nusselt number prediction although its effect on flow and temperature fields is remarkable for the case of four-fin arrangement.
(2) For all configurations, results indicate that local Nusselt number increases with an increase in Rayleigh number.
(3) Higher fin height has some blocking effect on fluid motion. If there is a tendency toward reducing heat transfer rate between two concentric horizontal cylinders, it is better to use higher fin height.
(4) Increasing the number of fins decreases mean Nusselt number ratio, with in turn reduces heat transfer rate.

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